Tachyons *

1. Causality Principle.

In section 31 we have shown that Lorentz transformations (9.13) are consistent with the constancy of the limiting velocity c in all inertial reference frames and the causality principle. The transformations (9.13) make sense for $V < c$ only (V being the velocity of the reference system $\mathcal{K}'$ with respect to $\mathcal{K}$) since at $V = c$ the denominators vanishes, and at $V > c$ they are imaginary. The causality principle requires the signal velocity be less than c. Therefore if the moving particle is regarded as a signal, its velocity v can not be greater than c. The analysis of the notion of signal, and of the process of emitting/sending and receiving signals, shows that the causality principle is of a thermodynamical nature and can be related to the second law of thermodynamics (the law of entropy increase) (this result is due to Y.P. Terletzky, Doklady Akad. Nauk SSSR, vol. 133, p. 329, 1960; see also [21]). In such a case the violation of the causality is admissible on the microscopic level, and on the macroscopic level too, but only as a random fluctuation.

After the above generalized comprehension of the causality principle (after 1960) a considerable number of papers and books appeared, dedicated to the possible existence of particles, moving faster than the light in the vacuum, and to their mechanics. Later the term *tachyons* has been adopted for such particles. For particles moving with subluminal velocities the term *bradyons* was coined, and particles moving with velocity of light c became known as *luxons*. The interest about tachyons increased in the last years in connection with the difficulties in explaining certain experimental results of nuclear processes accompanied with neutrino emission (e.g. β-decay of tritium $^3\mathrm{H} \longrightarrow {}^3\mathrm{He} + \mathrm{e}^- + \bar{\nu}_e$). The explanation could be possible, if one supposes that the *neutrino is a tachyon* with space-like 4-dimensional momentum (see e.g. the paper J. Ciborowski and J. Rembielinski, European Phys. Journal C, v. 8, p.

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*A further reading (sub)section of chapter 9 of the textbook: Dimitar Trifonov, *Classical Mechanics*. ISBN 954-9820-06-8, Sofia 2002 (in Bulgarian: Димитър Трифонов *Класическа механика*, София 2002).

157, 1999 and references therein). Results are reported about electromagnetic pulses with group velocity greater than c (see e.g. G. Kurizki et al, Opt. Spectrosc., v. 87, p. 505, 1999).

2. Lagrange function, energy and momentum of tachyon.

The tachyons can be described theoretically as relativistic particles with mass m and Lagrange function (LF) of the form

$$L = \mu c^2 \bar{\gamma}(v), \quad \bar{\gamma}(v) = \sqrt{v^2/c^2 - 1}, \tag{9.54}$$

where μ is a positive constant of mass dimension (so that L is of energy dimension), and v – the tachyon velocity, $v \geq c$. With this FL the action $S = \int Ldt$ is a relativistic invariant (scalar), since Ldt is invariant. Indeed, let us consider expression of the same form in $\mathcal{K}'$: $L'dt' = \mu c^2 \bar{\gamma}(v')dt'$. For simplicity we consider the motion along the OX axis and the special Lorentz transformations (9.13). From (9.13) we get the transformation of the interval dt and the particle velocity in the form

$$dt' = \frac{1 - vV/c^2}{\gamma(V)} dt, \qquad v' = \frac{v - V}{1 - vV/c^2}, \tag{9.55}$$

where, as before, $\gamma(V) = (1 - V^2/c^2)^{-1/2}$. The substitution of dt' and v' of (9.55) into the expression $\bar{\gamma}(v')dt'$ produces $\bar{\gamma}(v')dt' = \bar{\gamma}(v)dt$, wherefrom we get the required result: $L'dt' = Ldt$.

From the latter equality and (9.54) it follows that the momentum p and the energy E, defined by means of LF L,

$$\vec{p} = \frac{\partial L}{\partial \vec{v}} = \frac{\mu \vec{v}}{\sqrt{v^2/c^2 - 1}}, \quad E = \vec{p}\vec{v} - L = \frac{\mu c^2}{\sqrt{v^2/c^2 - 1}} \tag{9.56}$$

form, as in the case of bradyons, a four-dimensional vector $\underline{p}$ – the vector of the 4-dimensional *tachyon momentum*: $\underline{p} = (p^0, \vec{p})$, $p^0 = E/c$. This 4-vector however, unlike the bradyon momentum vector, *is not time-like* – it is, due to the inequality $v \geq c$, *a space-like* vector: from (9.56) we have

$$\underline{p}^2 := (p^0)^2 - \vec{p}^2 = -\mu^2 c^2. \tag{9.57}$$

In the relativity mechanics the rest mass m of any particle is defined through its squared four-dimensional momentum vector: $\underline{p}^2 = (p^0)^2 - \vec{p}^2 = m^2c^2$, i.e. $m^2 := \underline{p}^2/c^2$. Herefrom, taking into account (9.57), we get a negative value of the squared "rest mass" of the tachyon,

$$m^2 = -\mu^2 < 0,$$

i.e. the tachyon "rest mass" is *imaginary*: $m = i\mu$. However this is not a principal obstacle, since *the tachyons are never in the rest* – the tachyon velocity v is always greater than c. It is worth noting that v is not bounded from the above: at $v \longrightarrow \infty$ the tachyon energy, in accordance with (9.56), tends to null, and its momentum $|\vec{p}|$ tends to μc. At $v \longrightarrow c$ both the energy and momentum of tachyons, like the bradyon energy and momentum, tend to infinity.

3. Switching Principle.

Let in the reference frame $\mathcal{K}$ at time t_1 and point x_1 a tachyon is emitted with velocity v, which later at time $t_2 > t_1$ is absorbed at point x_2. Since $v > c$, it is possible to find a velocity V, such that in the frame $\mathcal{K}'$ the tachyon absorption occurs prior the time of emission, i.e. in $\mathcal{K}'$ the causality principle for the process "emission-absorption" of superluminal particle is violated. From eq. (9.55), rewritten for the finite time intervals in the form

$$\Delta t' = \frac{\Delta t}{\gamma(V)}(1 - vV/c^2), \tag{9.58}$$

we find that the above violation can occur if $c > V > c^2/v$, when the sign of the ratio $\Delta t'/\Delta t$ becomes negative. This shows that the tachyons can not be used as physical signals: in $\mathcal{K}'$, at $c > V > c^2/v$, the absorber in $\mathcal{K}$ becomes emitter, and the emitter becomes absorber. This switching from emitter to absorber and vice versa is advanced as a principle for particles with superluminal velocities – *the switching principle* [21] (for more details see [22] and e.g. the paper of G.D. Maccarrone and E. Recami, Nuovo Cimento A **57** (1980) 85).

Physical signals are used to transmit information from a given point and time to another point, i.e. we control the processes of emission and absorption. The switching principle states that with individual tachyons this is not possible.

4. Bradyon–tachyon symmetry. *

A natural question arises about how the tachyons look-like in reference frames $\mathcal{K}^*$ which move with superluminal velocities $V > c$. Let us call such frames tachyonic, or *superluminal reference frames*. We have readily to underline that $\mathcal{K}^*$ are hypothetical – we can not attach them to superluminal reference bodies, since such bodies are not known. Admitting the existence of superluminal frames we have to answer two main questions: a) what are the transitions from our subluminal $\mathcal{K}$ to (the superluminal) $\mathcal{K}^*$? b) what are the transitions from a given $\mathcal{K}^*$ to another $\mathcal{K}^{*\prime}$?

It turned out that the first problem has such a theoretical solution which leads to a remarkable symmetry between bradyons and tachyons in $\mathcal{K}$ and tachyons and bradyons in $\mathcal{K}^*$. Indeed, *assume* that the passage $\mathcal{K} \longrightarrow \mathcal{K}^*$ is described by the following time and coordinate transformation (here $V > c$),

$$\begin{aligned} ct^* &= \tfrac{1}{\bar\gamma(V)}(ct - Vx/c), \\ x^* &= \tfrac{1}{\bar\gamma(V)}(-Vt + x), \quad V > c, \end{aligned} \tag{9.59}$$

or, in a matrix form,

$$\underline{x}^* = \Lambda^*(V)\underline{x}, \quad \Lambda^* = \begin{pmatrix} 1/\bar\gamma(V) & -V/c\bar\gamma(V) \\ -V/c\bar\gamma(V) & 1/\bar\gamma(V) \end{pmatrix}.$$

This transformation is obtained from the special Lorentz transformation (9.13) with the replacement $\gamma(V) \longrightarrow \bar\gamma(V)$ and setting $V > c$. The matrices $\Lambda^*(V)$ obey the relations $\det\Lambda^* = -1$ and $\Lambda^{*T} g \Lambda^* = -g$. The latter means that they do not enter the Lorentz group (moreover, they do not form a group). These transformations convert the time-like 4-dimensional vectors into space-like vectors, and vice versa. In particular, the tachyon space-like 4-momentum in $\mathcal{K}$ is converted to a time-like vector in $\mathcal{K}^*$.

Let us show that the tachyon velocity $v > c$ in $\mathcal{K}$ becomes, for the observer in $\mathcal{K}^*$, less than c. Indeed, let x in eq. (9.59) be the coordinate of a particle moving with velocity $v = dx/dt$. Taking the differential of the left and right sides of the first equality in (9.59) we get

$$dt^* = \frac{1}{\bar\gamma(V)}(1 - vV/c^2)dt, \tag{9.60}$$

afterwhat, differentiating the second equality in (9.59), we find the relation between particle velocities in $\mathcal{K}$ and $\mathcal{K}^*$,

$$v^* := \frac{dx^*}{dt^*} = \frac{v - V}{1 - vV/c^2}. \tag{9.61}$$

The analysis of this relation shows, in view of $V > c$, that if $v \geq c$ (tachyon or luxon in $\mathcal{K}$), then $v^* \leq c$ (bradyon or luxon in $\mathcal{K}^*$), and vice versa – if $v \leq c$ (bradyon or luxon in $\mathcal{K}$), then $v^* \geq c$ (tachyon or luxon in $\mathcal{K}^*$). This symmetry is similar (but not identical) to the mirror symmetry – the more close to (or far below) c is the bradyon velocity v in $\mathcal{K}$, the more close to (or far above) c is the velocity v^* of its mirrored tachyon image in $\mathcal{K}^*$. Thus, *under the transition (9.59), the tachyons and bradyons are interchanged, the luxons remaining luxons.* The above picture does not contradict to the assumption, that the transitions from one superluminal reference frame $\mathcal{K}^*$ to another one $\mathcal{K}^{*\prime}$ be performed by the ordinary Lorentz transformations (9.13), which means that, for the observers in $\mathcal{K}^*$ (if $\mathcal{K}^*$ exist) the world of subluminal velocities would look like that for the observers in $\mathcal{K}$ (like our world). [Analyse yourselves how the two successive transformations $\Lambda^*(V_2)\Lambda^*(V_1)$ will act].

It is worth noting another property of the bradyon-tachyon transformations (9.59): From (9.60) one can see, that at $v = V$ one has $dt^* = -\bar{\gamma}dt$, i.e. the time interval dt^* (the proper time of tachyon) is of opposite sign with respect to dt. This means time inversion – the time in $\mathcal{K}^*$ flows in the inverse direction (compared to that in $\mathcal{K}$). The same is valid for the space intervals (lengths) dx^* and dx – at $v = V$ we have $dx^* = -\bar{\gamma}dx$, which means the inversion of the space coordinates. Let us again recall that the transformation (9.59) is only *a theoretical possibility*, establishing a mirror-like symmetry between tachyons and bradyons: in this "mirror" the tachyon looks like a bradyon, and the bradyon – like a tachyon. At $V \longrightarrow \infty$ the transformation (9.59) is reduced to the replacement of time and space axes with opposite signs: $ct \longrightarrow -x^*$, $x \longrightarrow -ct^*$.

Historical Notes.* The Special theory of relativity (STR), the basic notions and properties of which we have considered in the present chapter, is created with the efforts of many scientists, among which are (in historical order) H. Lorentz, A. Poincaré, A. Einstein and H. Minkowski. A decisive impetus to the investigations in the direction ot later formulated STR was given by the Michelson experimental result of 1881, and A. Michelson and E. Morley of 1887 (Amer. J. Sci. **3**, 34, 333), about the independence of light velocity from the source motion. An extensive list of references on STR can be found in [23].

The Lorentz transformation are published by H. Lorentz in 1904 (Proc. Acad. Sci., Amsterdam, 1904, v. 6, p. 809 (Russian translation in [23]) as transformations that leave the Maxwell equations of electromagnetic field invariant. In the form (9.13) they are written by A. Poincaré (Comptes Rendues **140**, 1504 (1905)), who called them "Lorentz transformations"and derived the law of velocity addition formula (9.18) (the transformations (9.59): $\mathcal{K} \longrightarrow \mathcal{K}^*$, for all I know, are not considered). However Lorentz, unlike Einstein in 1905, does not regard the quantities x', y', z' and t' as physical space coordinates and time. Einstein (Annalen der Physik **17**, 891 (1905)), obtains Lorentz transformations from the following two postulates (in essence this derivation is given in most of the textbooks, see e.g. [21]):

1. The laws, according to which the states of physical systems are changing in time, do not depend on the fact to which of two coordinate systems performing uniform translation one with respect to another, these state changes are referred.

2. Every light ray is moving with definite velocity V in the "rest frame independently of the fact from a rest or from a moving body this light ray is emitted.

In his talk in the Saint Louis in 1904 (published in "Bulletin des Science Mathematiques" **28**, ser. 2, 302 (1904)) A. Poincaré formulates the relativity principle in the following manner: "... the principle of relativity, according to which the laws of physical phenomena must be the same for the observer in rest and for the observer, moving with constant velocity". About the light velocity limit Poincaré writes in the same article provisionally: "Based on all of these results, if they will be confirmed, a completely new mechanics would arise, which will be characterized first of all with the property that no velocity could be greater than the velocity of light".

From the Lorentz transformations (9.13) it follows that, as we have shown in the main text, there is a change in the time and space intervals. But ideas and even formulae for such changes are expressed previously by Fitzgerald (1891: $l = l_0\sqrt{1-v^2/c^2}$), Lorentz (1892) and Poincaré (1902: "The absolute space and absolute time do not exist"). The pseudo Euclidean structure of the space of events (the relativistic interval, the terms 'proper time', 'world point', and 'world line') is introduced by H. Minkowski in 1907-1908 publications (see e.g. [23]). The Lagrange function for relativistic particle (9.37) is introduced by Plank (M. Planck, Verhandl. Deutsch. Phys. Ges., b. 4, s. 136, 1906 (Russian translation in [23])).

It is worth noting (noted also in 1908 by H. Minkowski [23], and e.g. in [21]) that, transformations of the form similar to the Lorentz transformations (9.13) are obtained earlier by W. Foigt (Gött. Nachr., b. 5, s. 41, 1887) in a paper on the Doppler effect. The well known as Einstein formula $E = mc^2$ relating the particle mass and energy also has an interesting history, which is described e.g. in the Fadner paper (W.L. Fadner, American Journal of Physics, **56**, 114 (1988) [Bulgarian translation: Svetat na fizikata **24** (3), 218 (2001)] .

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